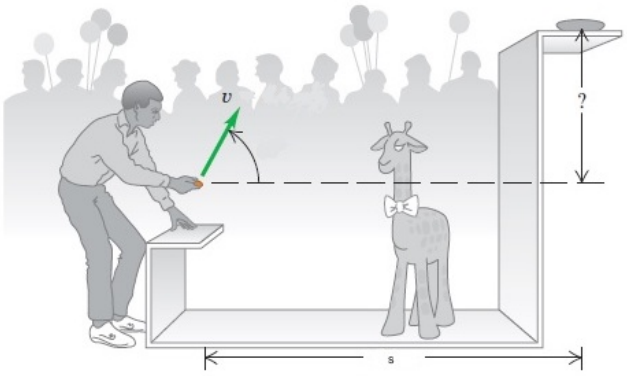
Physics 1A
Spring 2018

Midterm Exam 1 Solutions

Friday, April 27, 2018
16:00 – 16:50, PAB 1-725

Problem 1

In a carnival booth, you win a stuffed giraffe if you toss a quarter into a small dish. The dish is on a shelf above the point where the quarter leaves your hand and is a horizontal distance of s meters from this point (see figure). If you toss the coin with a velocity of $\vec{v}$ at an angle of α degrees above the horizontal, the coin lands in the dish. You can ignore air resistance.
a) What is the height of the shelf above the point where the quarter leaves your hand?
b) What is the vertical component of the velocity of the quarter just before it lands in the dish?



Solution 1

a) First we decompose the velocity into its x- and y-direction components:

$$v_x = |\vec{v}| \cos\alpha \ , \qquad v_y = |\vec{v}| \sin\alpha \ . \tag{1}$$

The time it takes the quarter to travel the horizontal distance s is given by

$$t = \frac{s}{v_x} = \frac{s}{|\vec{v}| \cos\alpha}, \tag{2}$$

and then we know that this is the same time that is needed to go the vertical distance h:

$$h = v_y t - \frac{1}{2} g t^2 \ . \tag{3}$$

We just substitute t and v_y to get

$$h = |\vec{v}| \sin\alpha \frac{s}{|\vec{v}| \cos\alpha} - \frac{1}{2} g \frac{s^2}{|\vec{v}|^2 \cos^2\alpha} = s \tan\alpha - \frac{g s^2}{2|\vec{v}|^2 \cos^2\alpha}. \tag{4}$$

b) Easily,

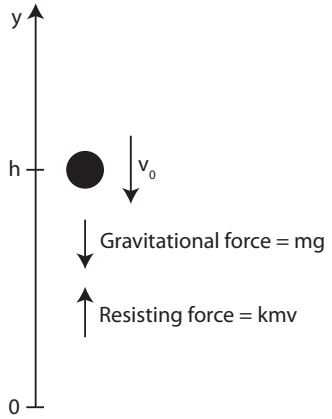
$$v(t) = v_{y,0} - gt = |\vec{v}| \sin\alpha - gt = |\vec{v}| \sin\alpha - g \frac{s}{|\vec{v}| \cos\alpha}. \tag{5}$$

Problem 2

A particle of mass m is falling vertically in a gravitational field. The retarding force applied on the particle is *proportional* to its velocity. Given that the particle's initial position and velocity (at $t = 0$) are h and v_0, find the velocity and displacement (at $t \neq 0$). You might find the following integrals useful: $\int \frac{1}{a+x} dx = \ln(a + x) + \text{const.}$ and $\int e^{ax} dx = \frac{1}{a} e^{ax} + \text{const.}$

Solution 2

The particle is falling downwards with an initial velocity v_0 from a height h in a gravitational field as shown in the figure below.



$$F = m \frac{dv}{dt} = -mg - kmv. \tag{6}$$

Note that $v < 0$, therefore, $-kmv > 0$.

$$\frac{dv}{dt} = -g - kv \rightarrow \frac{dv}{-g - kv} = dt \rightarrow \frac{dv}{g + kv} = -dt \rightarrow \frac{1}{k} \ln(kv + g) = -t + c \tag{7}$$

At $t = 0$, $v = v_0$, therefore $c = \frac{1}{k}\ln(kv_0 + g)$. Hence,

$$v = -\frac{g}{k} + \frac{kv_0 + g}{k}e^{-kt} \tag{8}$$

Now, we can find the displacement

$$\frac{dy}{dt} = v \to y = -\frac{g}{k}t + \frac{kv_0 + g}{k^2}e^{-kt} + c \tag{9}$$

At $t = 0$, $y = h$. Therefore, $c = h - \frac{kv_0+g}{k^2}$. Hence,

$$y = h - \frac{g}{k}t + \frac{kv_0 + g}{k^2}\left(e^{-kt} - 1\right) \tag{10}$$

Problem 3

A car of mass m is traveling around a flat circular race track of radius R. The static coefficient of friction between the tire and the road (against transverse motion) is μ. (a) How fast can the car travel before it starts to skid? (b) What is the angular velocity ω of the car at the speed calculated in (a).

Solution 3

(a) The max speed of the car should satisfy $mv^2/r = \mu mg$. We find

$$v = \sqrt{\mu g r}. \tag{11}$$

(b) The angular velocity is $\omega = v/r$.